An Overview of Membrane Computing

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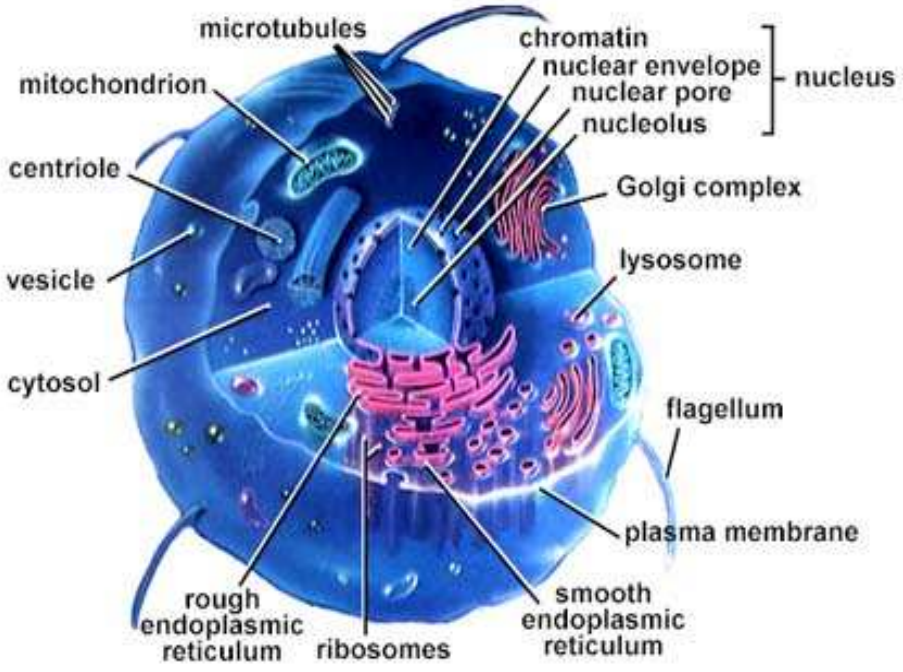
Membrane Computing

"The paradigmatic idea of membrane computing is to see whether we can mimic the living cell - its structure and functioning"

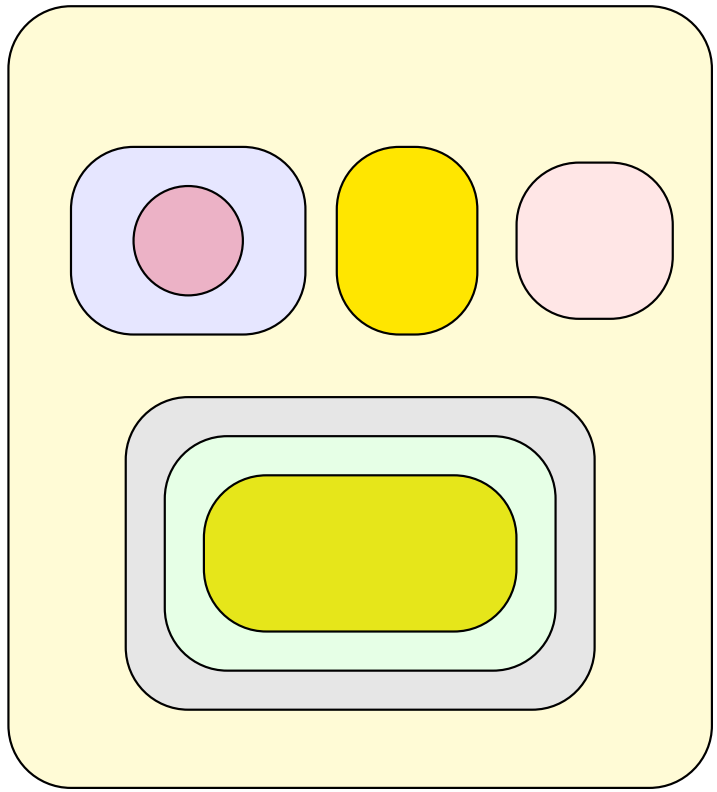
Gheorghe Păun

The Living Cell



What a jungle!

Abstraction of the Living Cell



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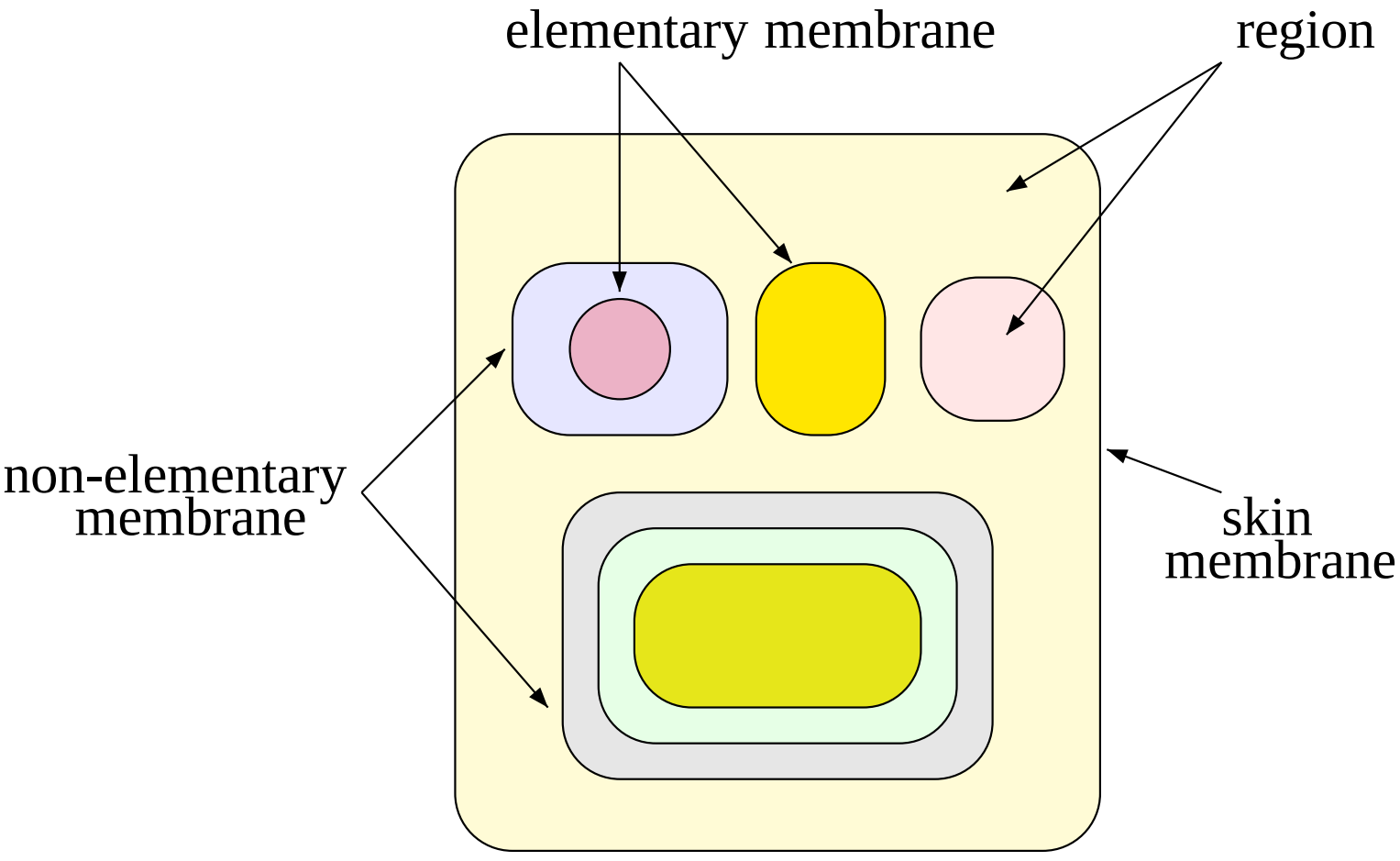

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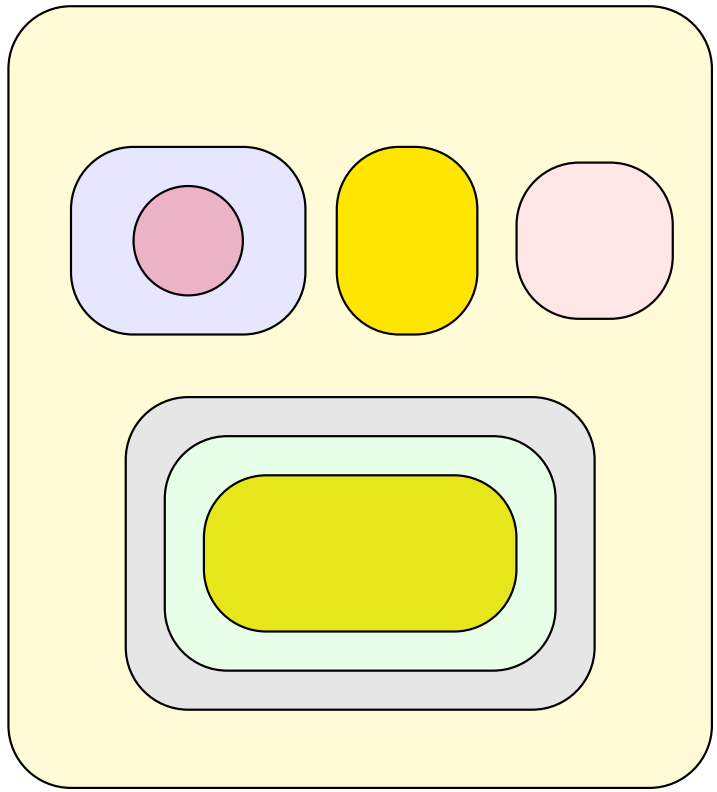

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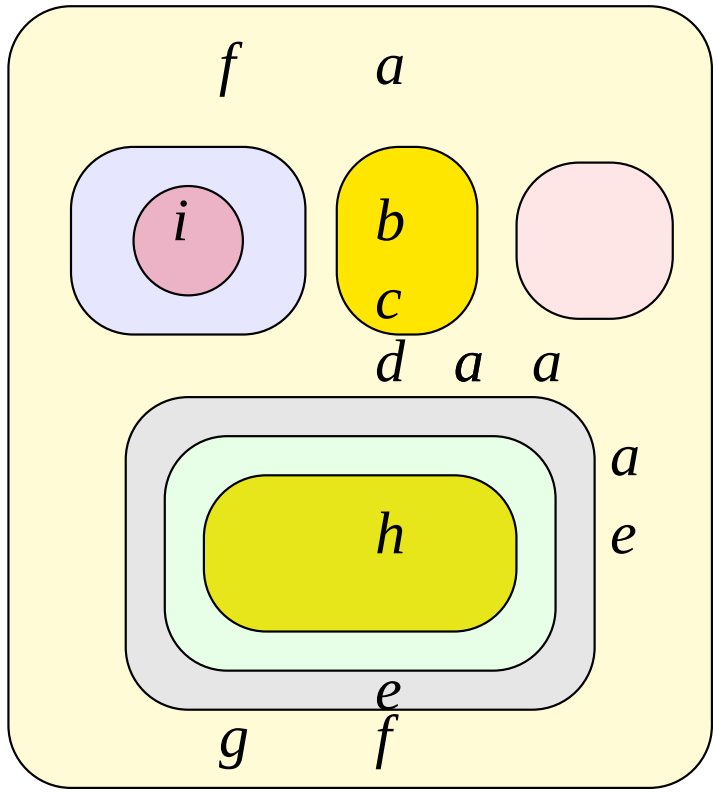

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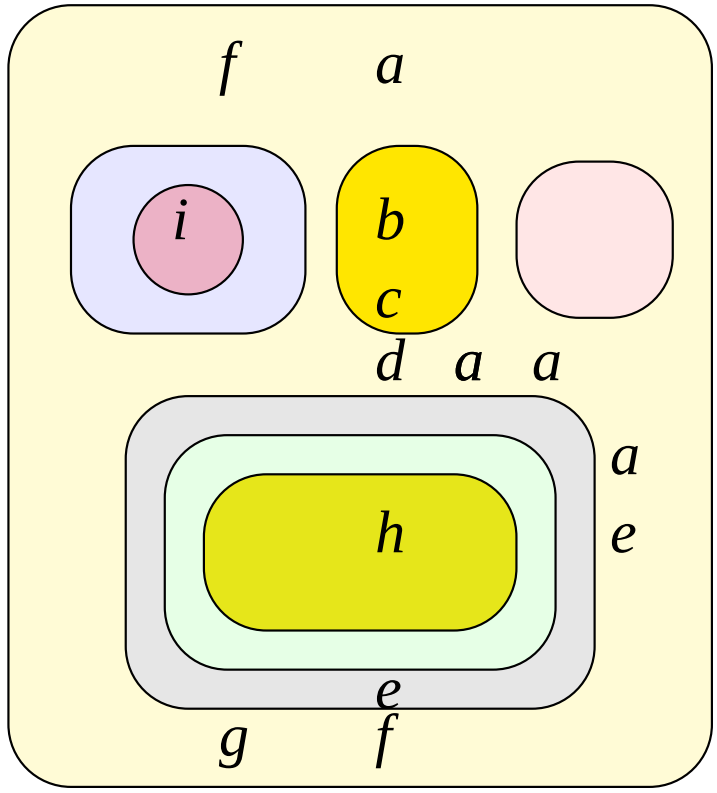


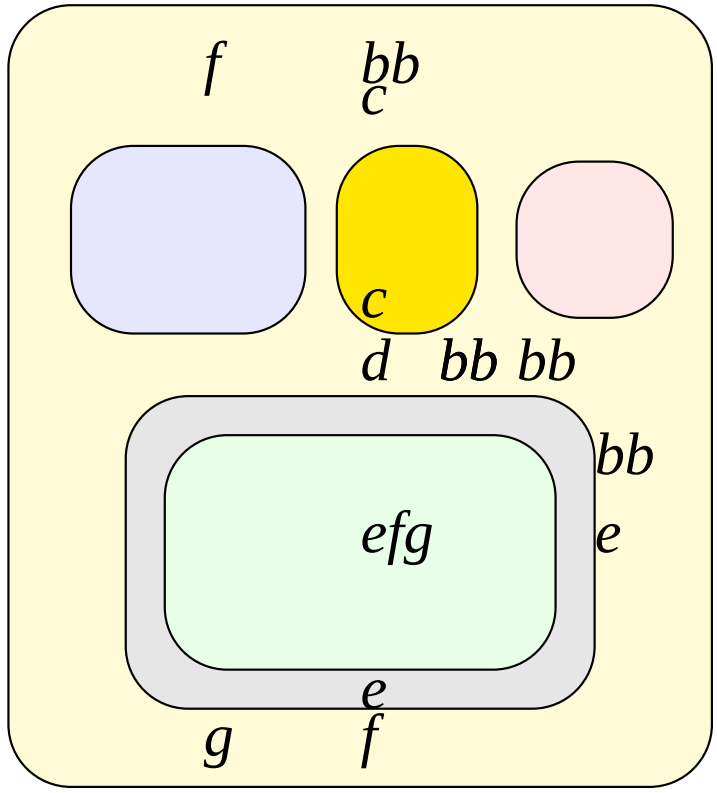
$a \rightarrow bb$

$b \rightarrow (c, out)$

$i \rightarrow \delta$

$h \rightarrow efg\delta$

Abstraction of the Living Cell



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Basic Ingredients

- non-deterministic choice of rules, objects
- maximal parallelism
- transitions, computation, halting
- internal output, external output

Result : Cell-like abstract system : P System

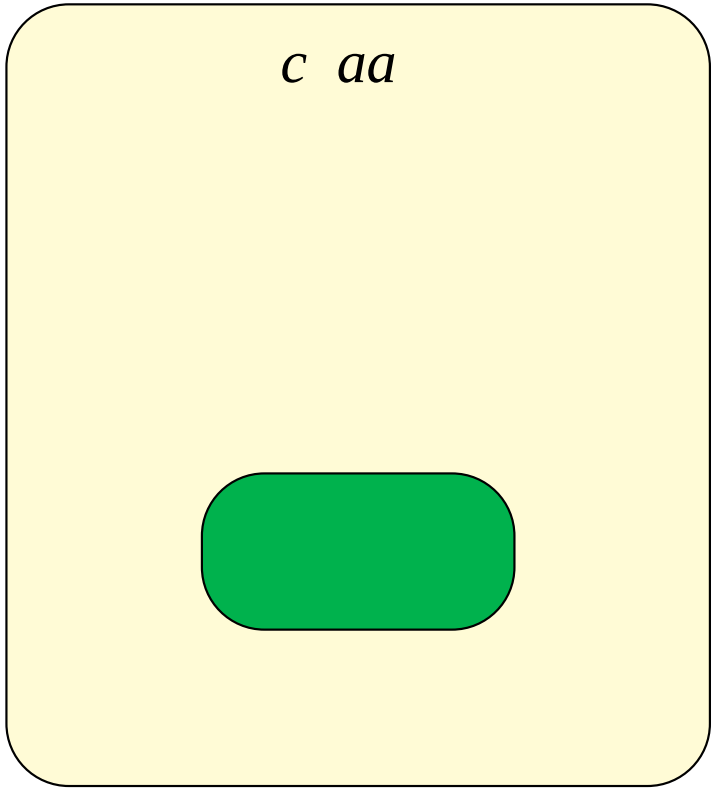
Basic Ideas in Membrane Computing

Symbol Objects (as in the example)

- $\Pi = (O, \mu, w_1, \ldots, w_m, R_1, \ldots, R_m, i_0)$
- μ : membrane structure
- w_i : multisets of objects in membrane i
- R_i : rules of the form $a \to (u, in)(v, out)w$ or $ca \to c(u, in)(v, out)w$, c : catalyst
- i_0 : output membrane

Example : Symbol Objects

Using catalysts, promoters. Input : a^n; Output : e^{n^2}



$$a \longrightarrow b_1 b_2$$
$$cb_1 \longrightarrow cb_1'$$
$$b_2 \longrightarrow b_2(e, in)/b_1$$

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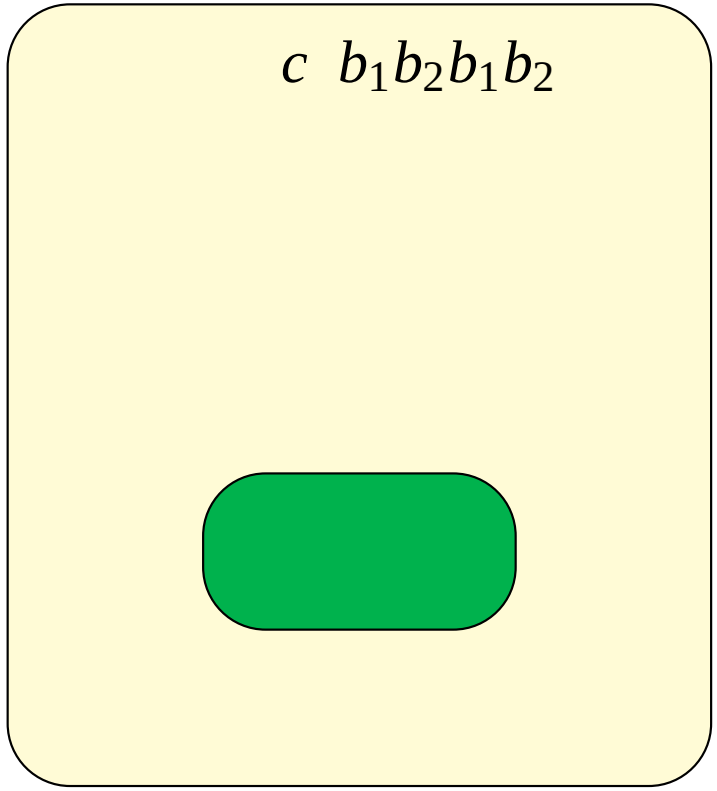

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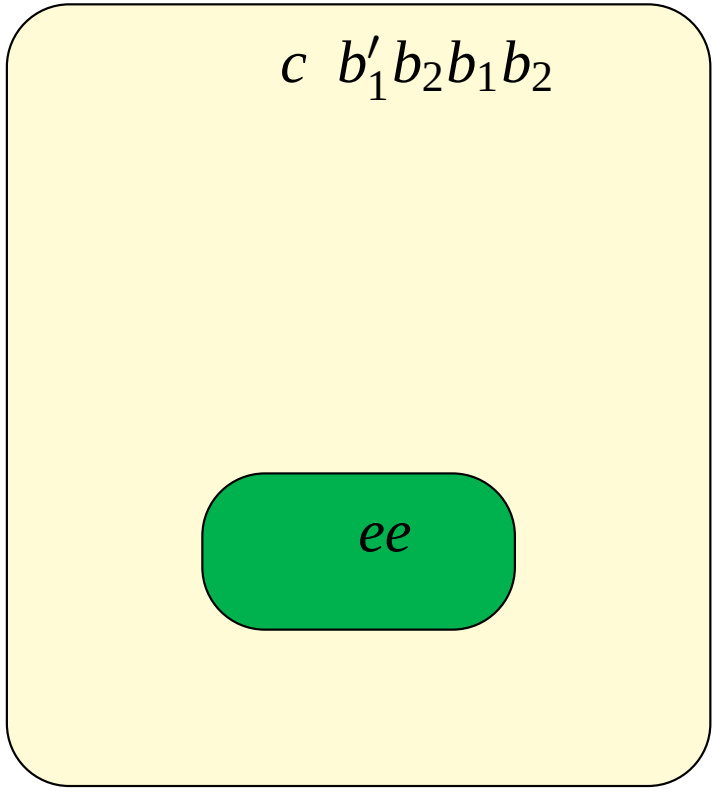

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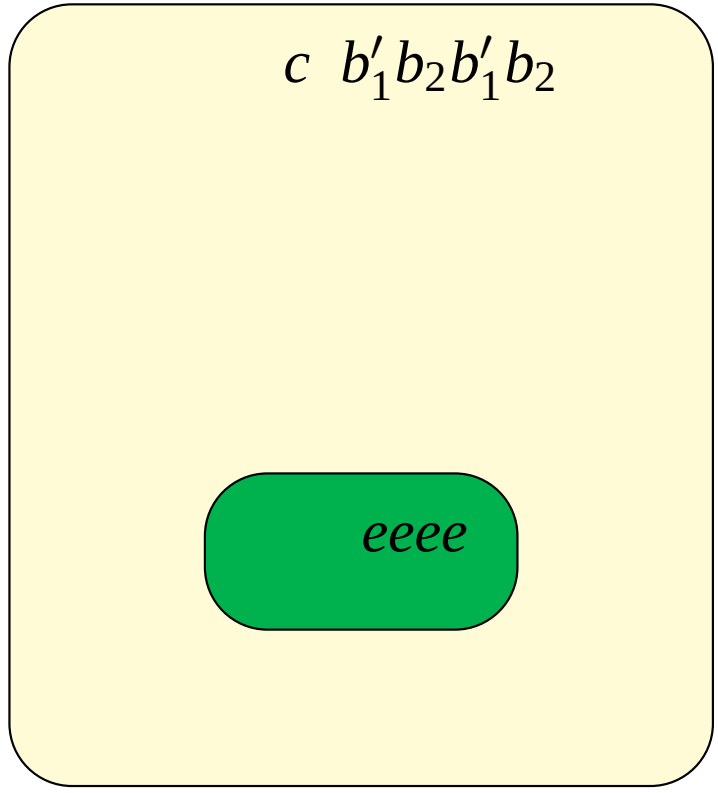

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Computational Power

Families $\mathbb{N}OP_m(\alpha)$, $\alpha \in \{coo, ncoo, cat\} \cup \{cat_i \mid i \geq 1\}$, $m \geq 1$ or $m = *$

Lemma

(collapsing hierarchy) $\mathbb{N}OP_*(\alpha) = \mathbb{N}OP_m(\alpha)$, $\alpha \in \{coo, ncoo, cat\}$ *and* $m \geq 2$.

Theorem

$\mathbb{N}OP_*(ncoo) = \mathbb{N}OP_1(ncoo) = \mathbb{N}CF$

Theorem

$\mathbb{N}OP_2(cat_2) = \mathbb{N}RE$

Conjecture: $\mathbb{N}RE - \mathbb{N}OP_*(cat_1) \neq \emptyset$

String Objects

- rewriting rules
- inspired by splicing and (other) DNA operations
- More complex objects : arrays, pictures

Computing by communication : symport/antiport

- $(x, in); (x, out)$: symport; $(x, in; y, out)$: antiport
- $max(|x|, |y|)$: weight
- System : $\Pi = (O, \mu, w_1, \ldots, w_m, E, R_1, \ldots, R_m, i_0)$
- $E \subseteq O$ is the set of objects that appear in the environment in arbitrarily many copies
- Families $\mathbb{N}OP_m(symp_p, anti_q)$

Theorem

$\mathbb{N}RE = \mathbb{N}OP_1(sym_0, anti_2) = \mathbb{N}OP_2(sym_2, anti_0) =$
$\mathbb{N}OP_1(sym_3, anti_0) = \mathbb{N}OP_3(sym_1, anti_1)$

Active Membranes

$a[\,]_i \rightarrow [b]_i$	go in
$[a]_i \rightarrow b[\,]_i$	go out
$[a]_i \rightarrow b$	dissolution
$a \rightarrow [b]_i$	membrane creation
$[a]_i \rightarrow [b]_j[c]_k$	membrane division
$[b]_j[c]_k \rightarrow [bc]_l$	membrane merging
$[u]_i \rightarrow [\,]_i[u]_j$	gemmation
$[Q]_i \rightarrow [O-Q]_j[Q]_k$	separation

Applications: Efficient solutions of intractable problems
(illustration coming up later)

Some More Variants

- Tissue-like systems : membranes in the nodes of a graph
- P automata
- Systems with objects on membranes : inspired by brane calculi
- Spiking neural systems
- P colonies : set of cells of a bounded capacity, with minimal object processing rules
- Metabolic Systems
- Mobile Membranes

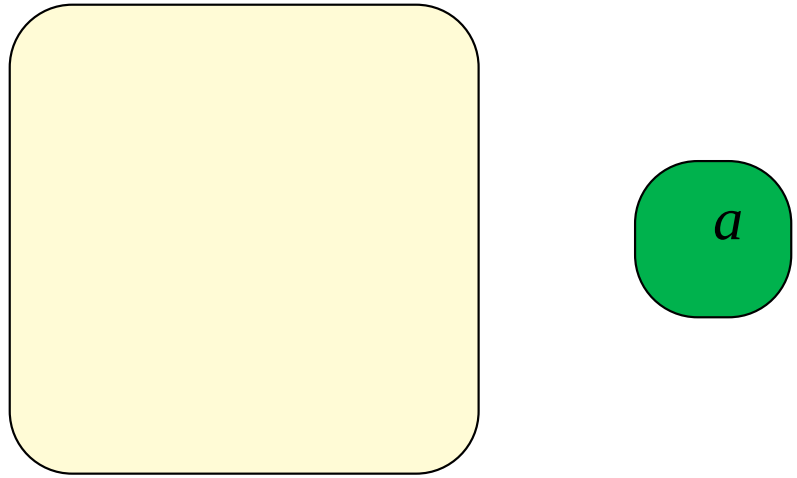
Results

- Characterization of Turing completeness ($RE, \mathbb{N}RE, PsRE$)
- Comparison with Chomsky hierarchy, L-systems
- Polynomial time solutions to NP-complete/PSPACE-somplete problems (using an exponential workspace created in a biological way)
- Other types of mathematical results (normal forms, hierarchies, determinism versus non-determinism, complexity)
- Connections with ambient calculus, petri nets, X-machines, quantum computing, brane calculus
- Simulations and implementations
- Applications

Turing Completeness

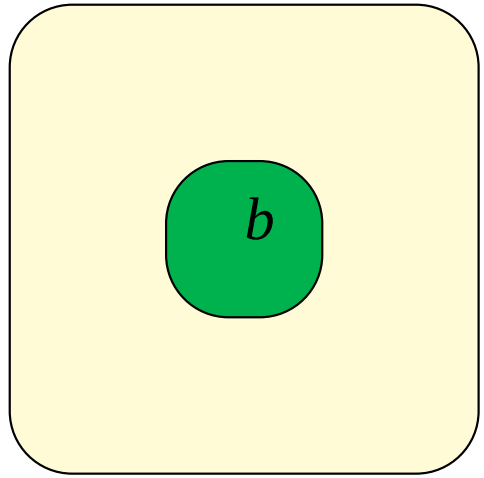
Mobile Membranes

Endocytosis



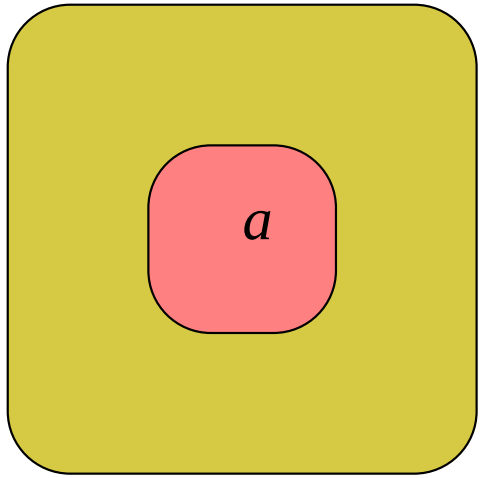
Mobile Membranes

Endocytosis



Mobile Membranes

Exocytosis



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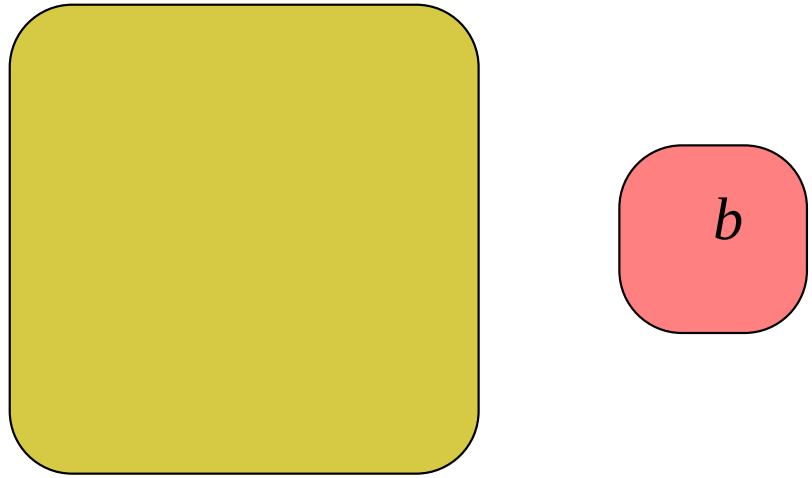
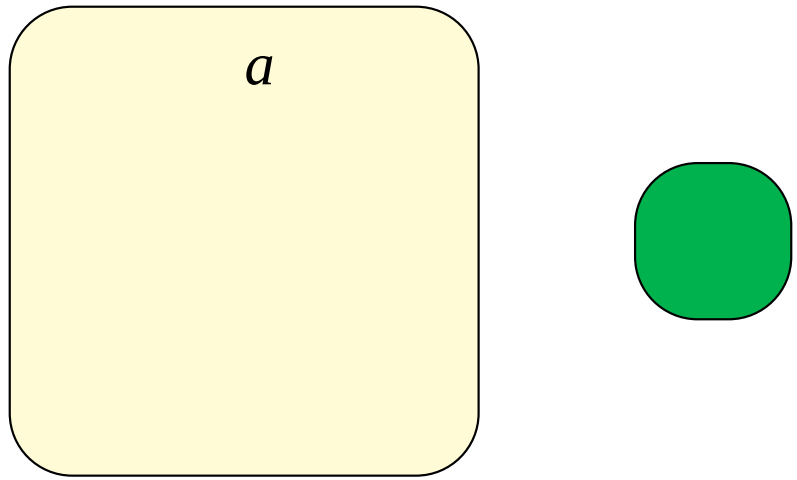


Mobile Membranes

Forced Endocytosis



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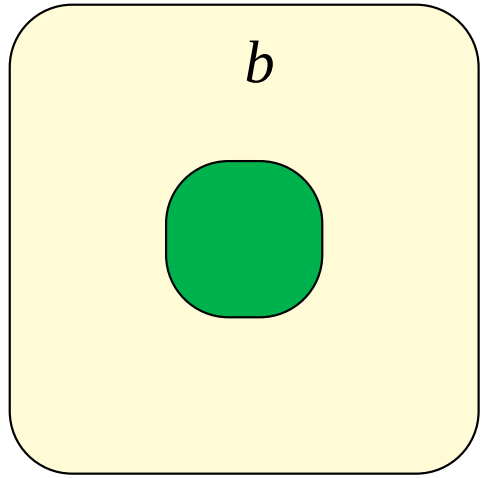

Mobile Membranes

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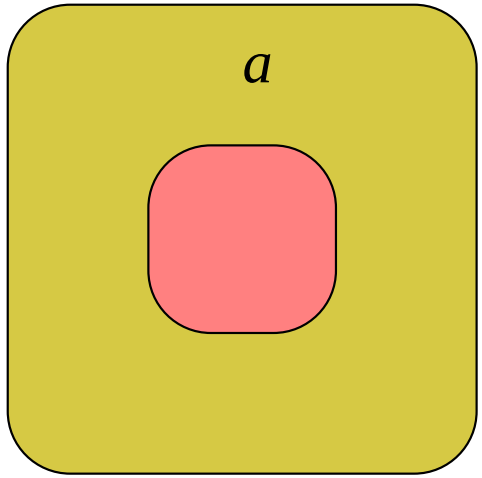
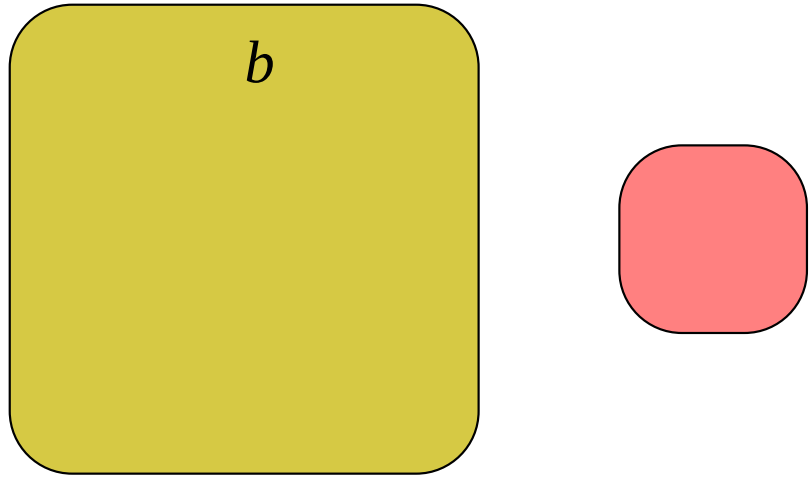


Mobile Membranes

Forced Exocytosis



Mobile Membranes

$[a]_h[\ ]_m \rightarrow [[w]_h]_m$	endocytosis	(*endo*)
$[[a]_h]_m \rightarrow [w]_h[\ ]_m$	exocytosis	(*exo*)
$[\ ]_h[a]_m \rightarrow [[\ ]_h w]_m$	forced endocytosis	(*fendo*)
$[a[\ ]_h]_m \rightarrow [\ ]_h[w]_m$	forced exocytosis	(*fexo*)

Mobile Membranes are Turing Complete

Families $\mathbb{N}EM_n(\alpha)$, $\alpha \subseteq \{exo, endo, fendo, fexo\}$

Theorem

$\mathbb{N}EM_5(endo, exo, fendo, fexo) = \mathbb{N}RE$

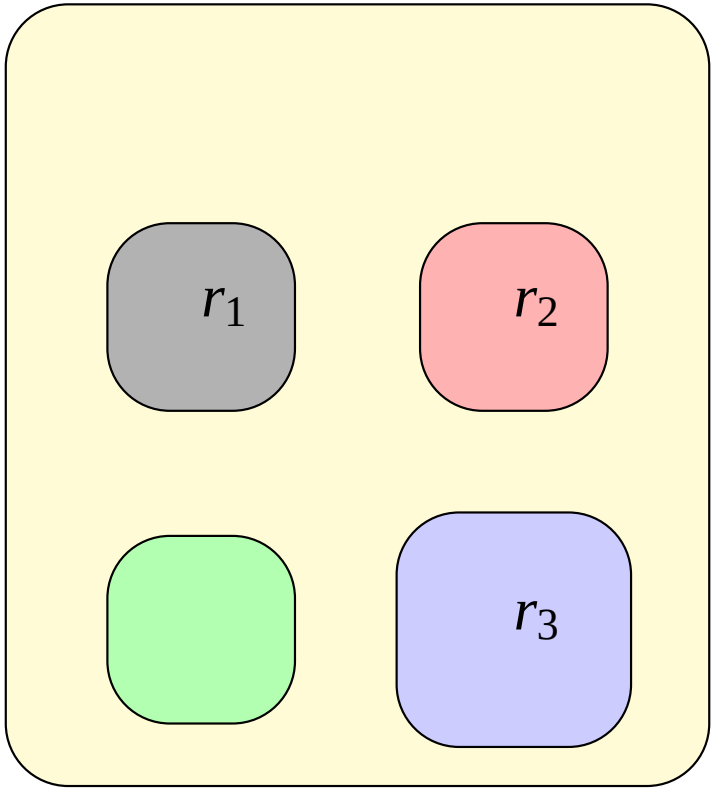
- Proof Idea: Simulate a register machine with 3 registers
- Register machine : (n, P, i, h)
  - n : number of registers
  - P : program with labeled instructions of the form $(add(r), k, l)$ or $(sub(r), k, l)$
  - i : initial instruction; h: final instruction

Theorem

If $L \subseteq V^*$, $card(V) = k, L \in RE$, then a 3-register machine M exists such that for every $w \in V^*$ we have $w \in L$ if and only if M halts when starting with $val_{k+1}(w)$ in its first register; in the halting step, all registers of the machine are empty.

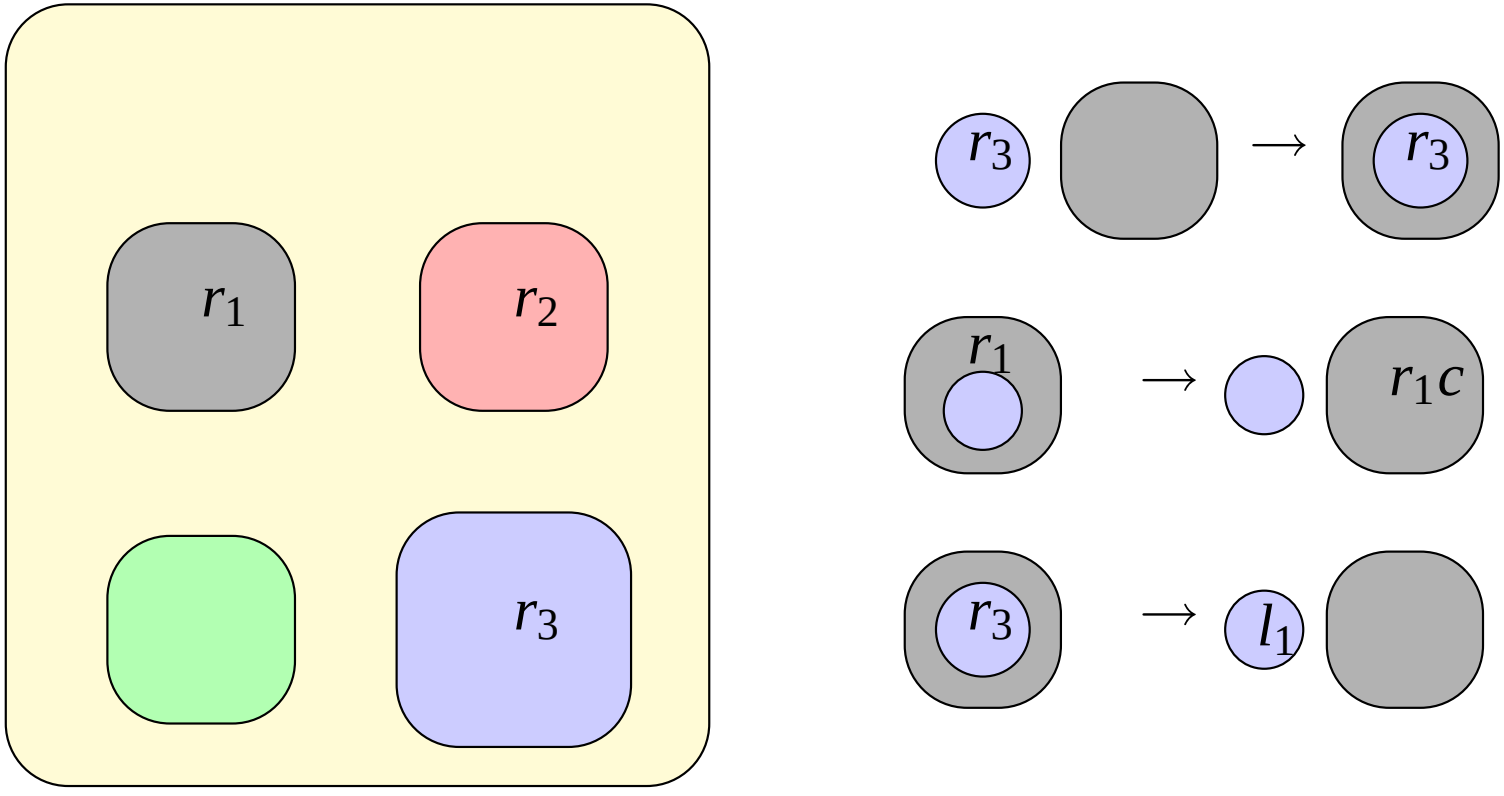
Proof Details

Initial configuration



Proof Details

Generation of Initial contents of register 1:



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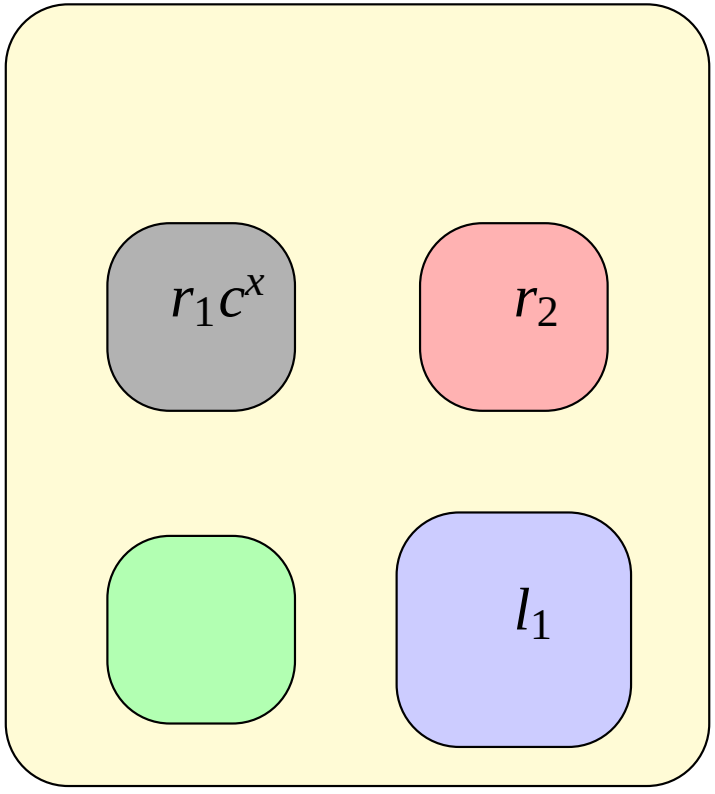
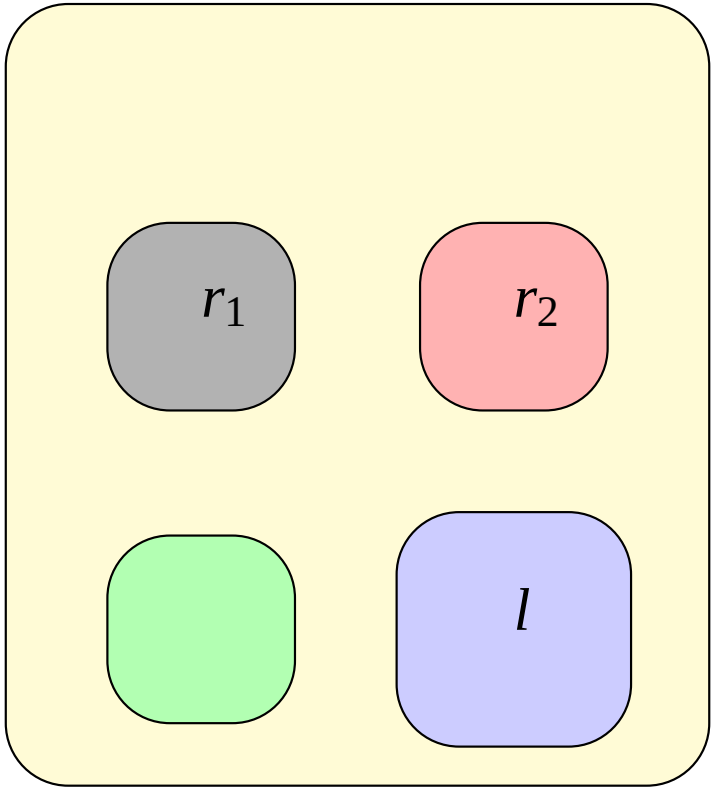


Simulation of a decrement instruction

$l : (sub(1), j, k)$. Register 1 empty.



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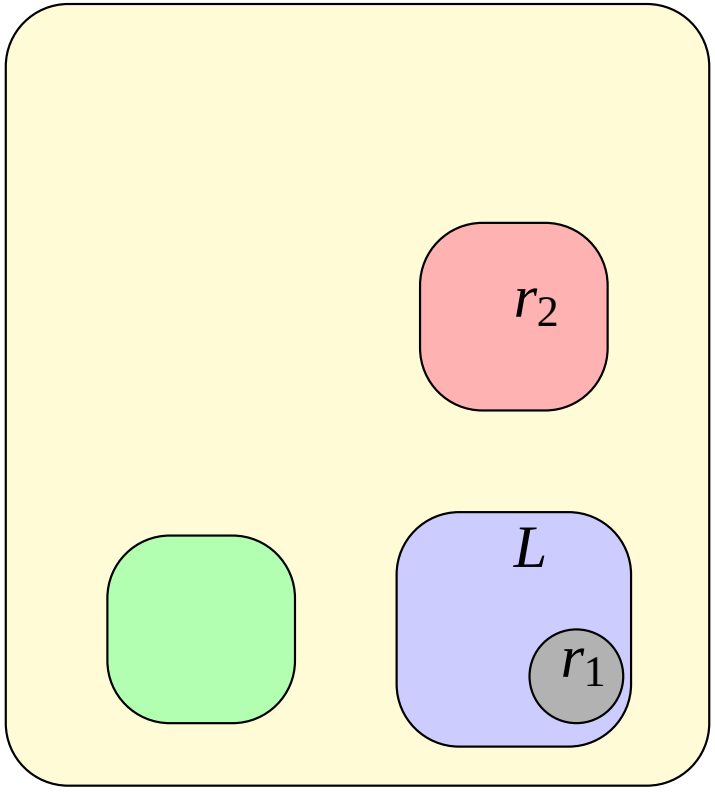

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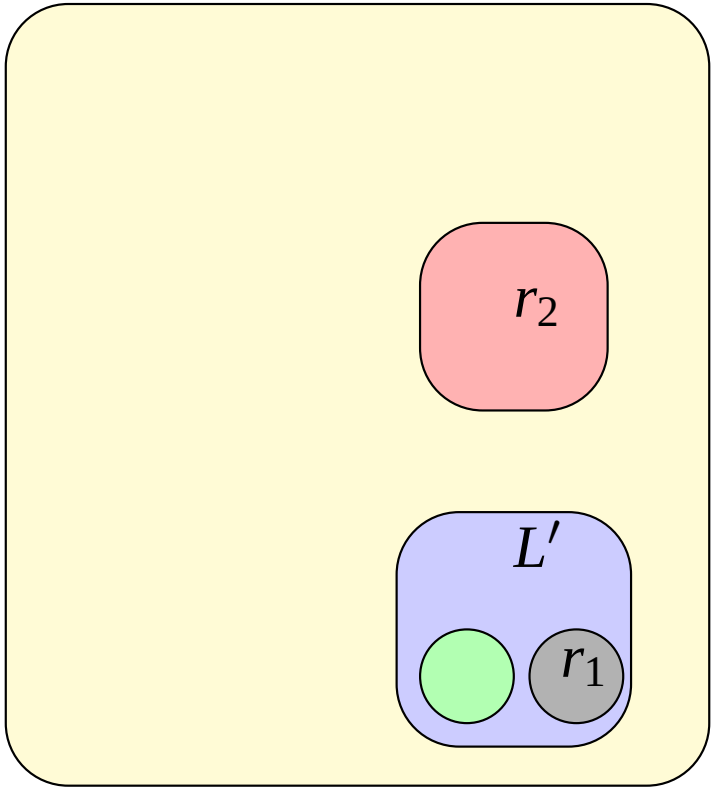

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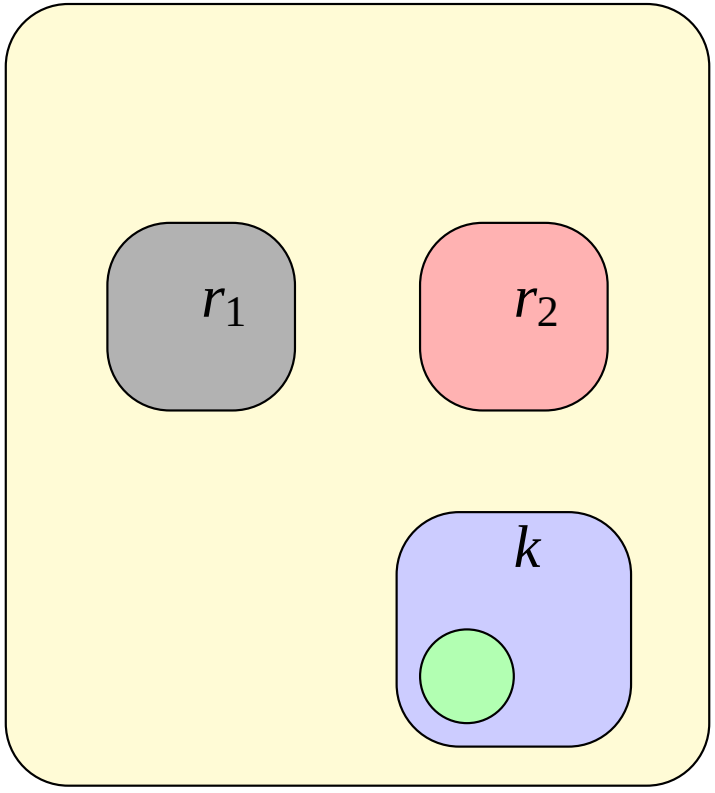

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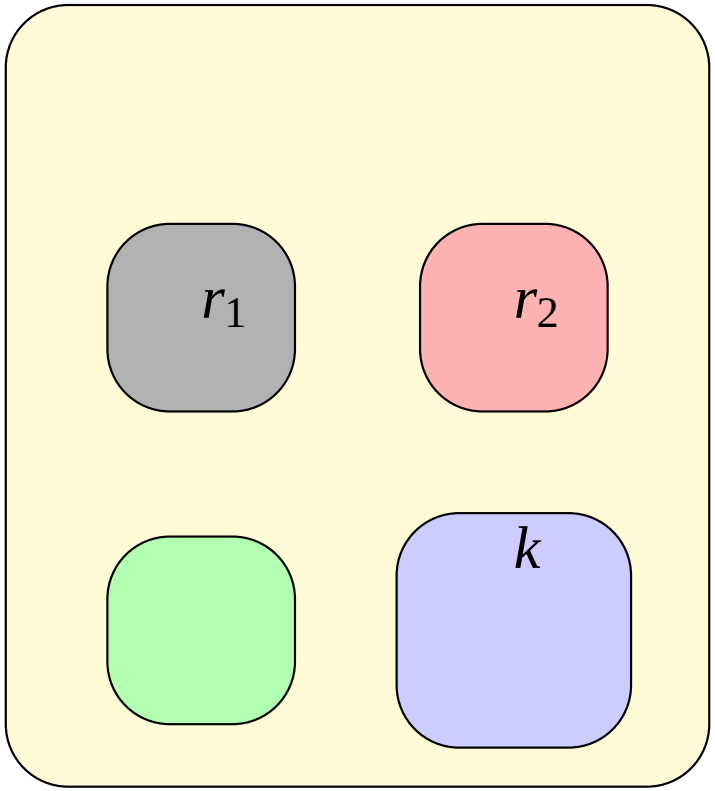

Simulation of a decrement instruction

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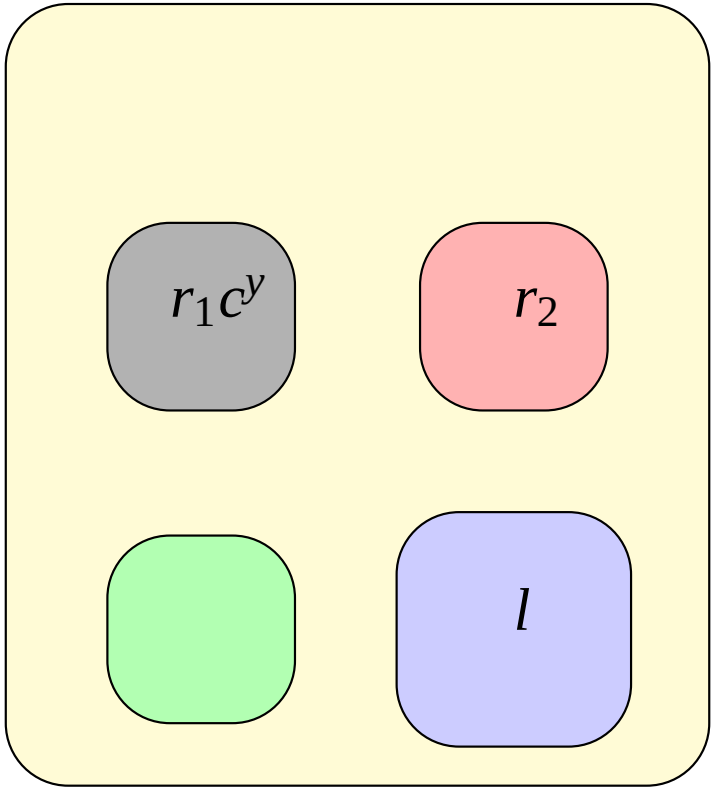

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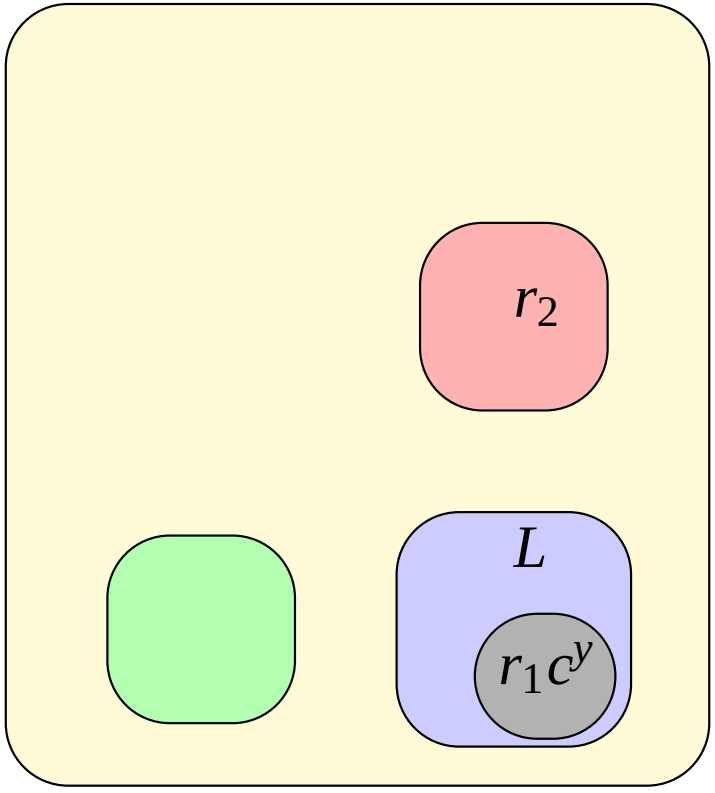

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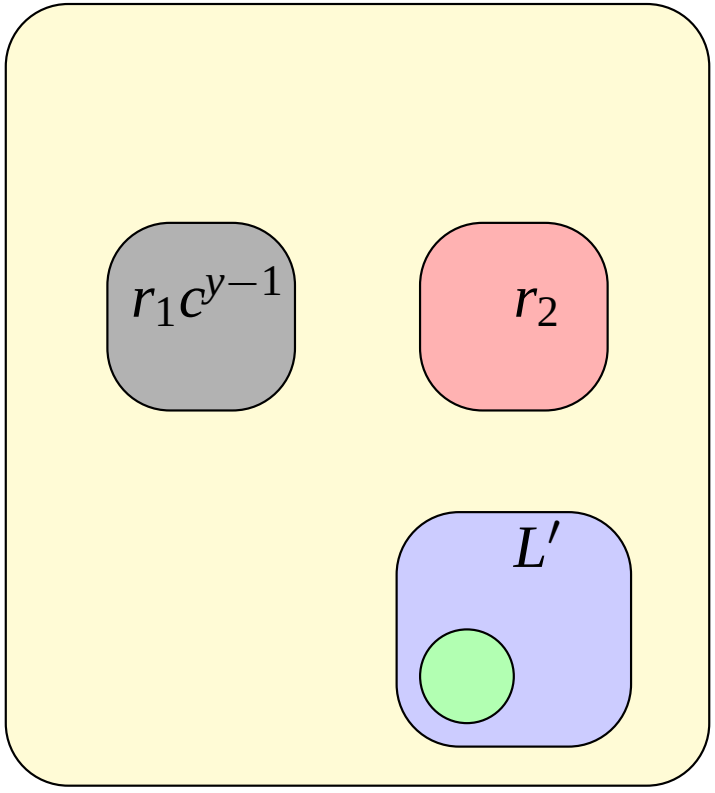

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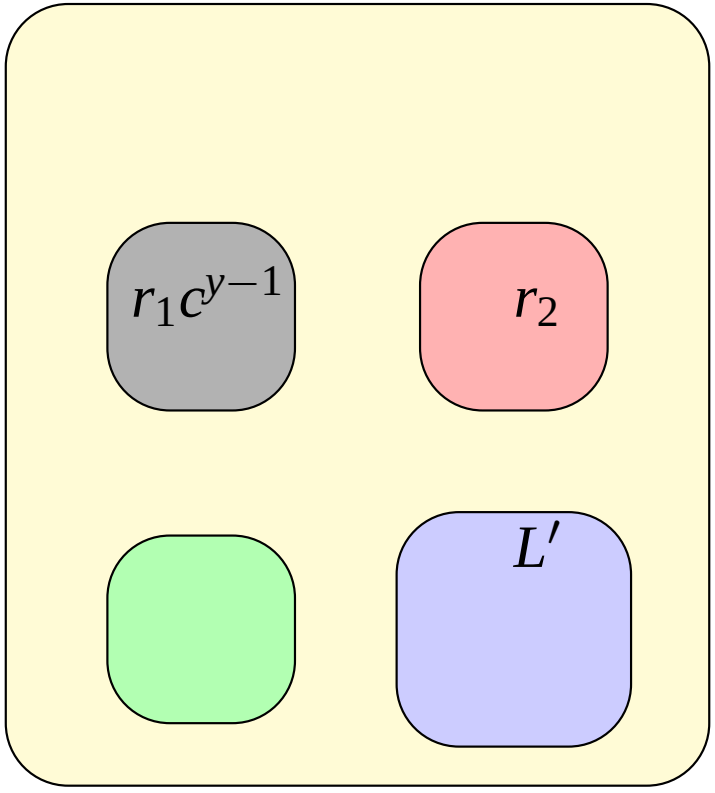

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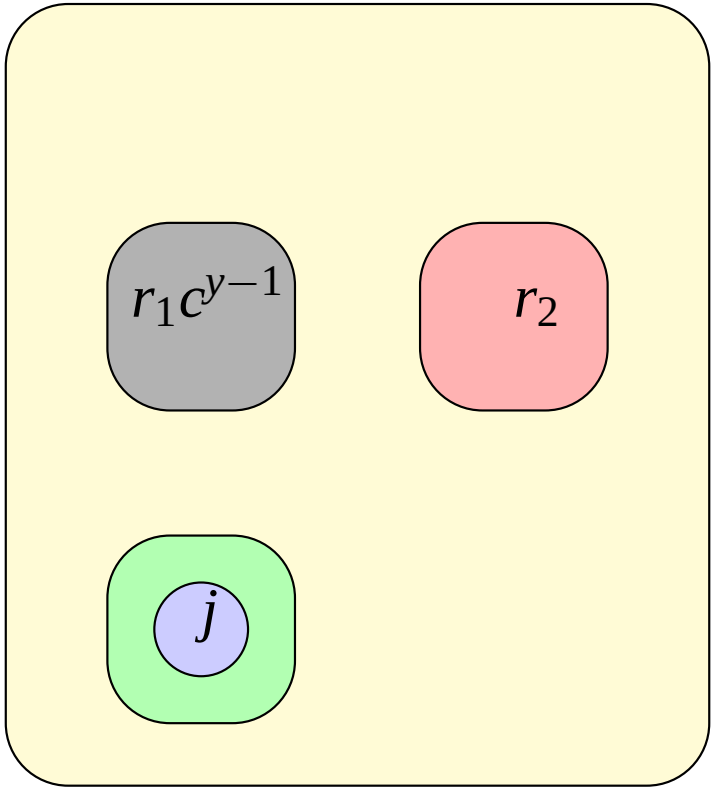

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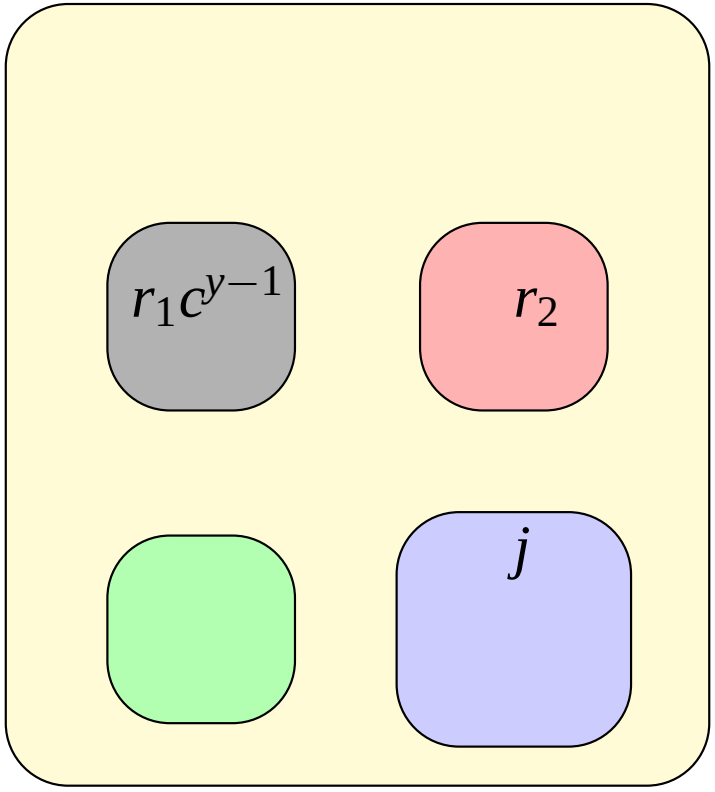


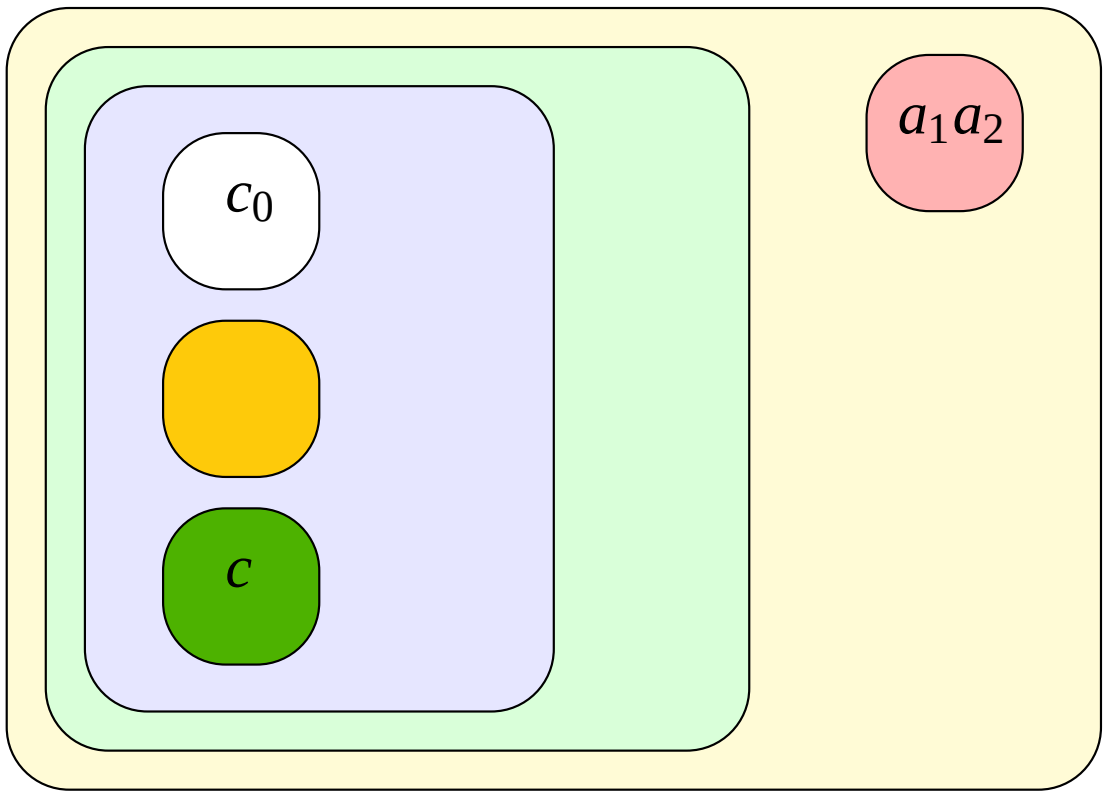
An Application : Uniform Solution to SAT

Solving SAT

- $\varphi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ in CNF
- Variables $\{x_1, \ldots, x_n\}$
- C_i of the form $y_1 \vee y_2 \vee \cdots \vee y_r$, $r \leq n$, $y_i = x_i$ or $\neg x_i$
- Propose a polynomial time uniform solution : for all formula instances of size (n, m), takes time $\mathcal{O}(m + n)$
- Uses operations $endo, exo, div$

Example

$\varphi = (x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_2)$. Solution : $\{TT, FF\}$



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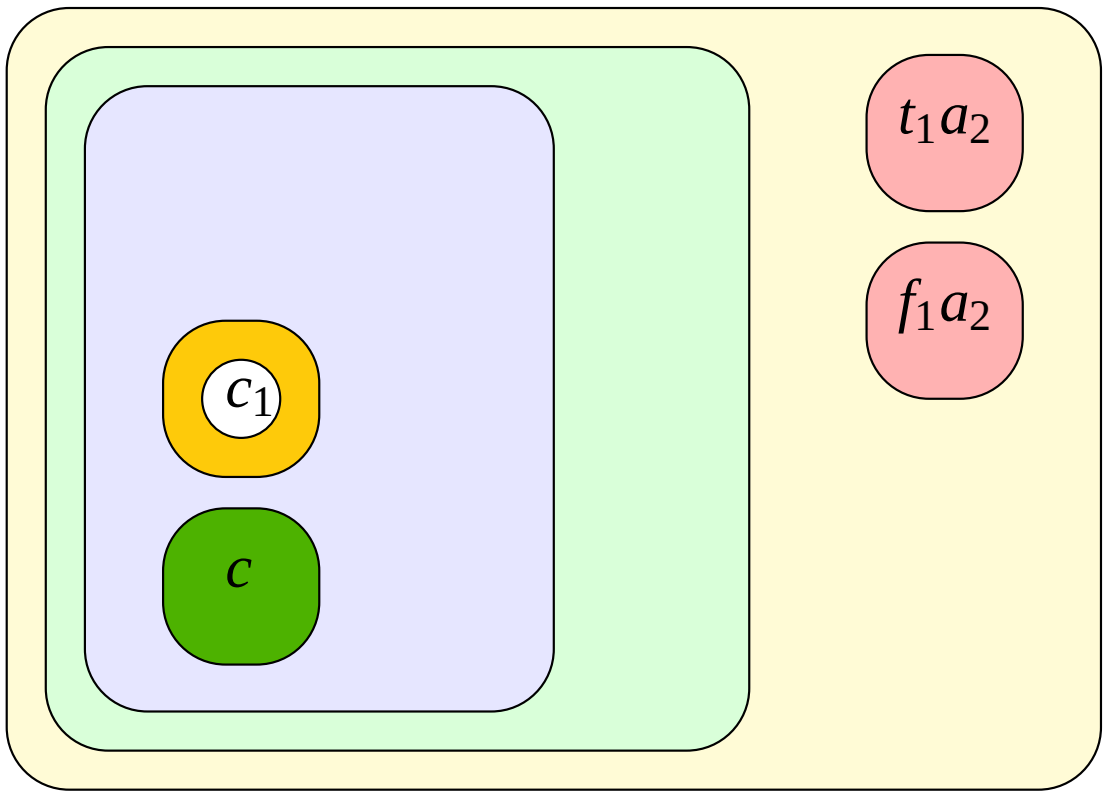

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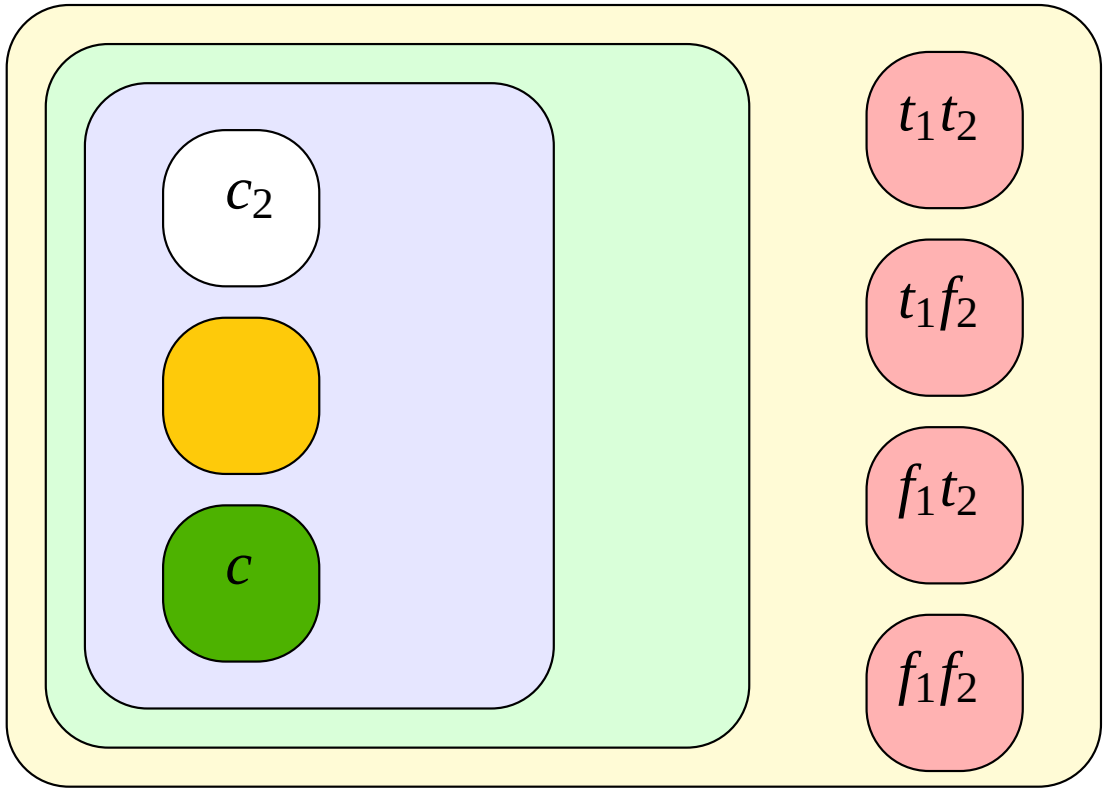

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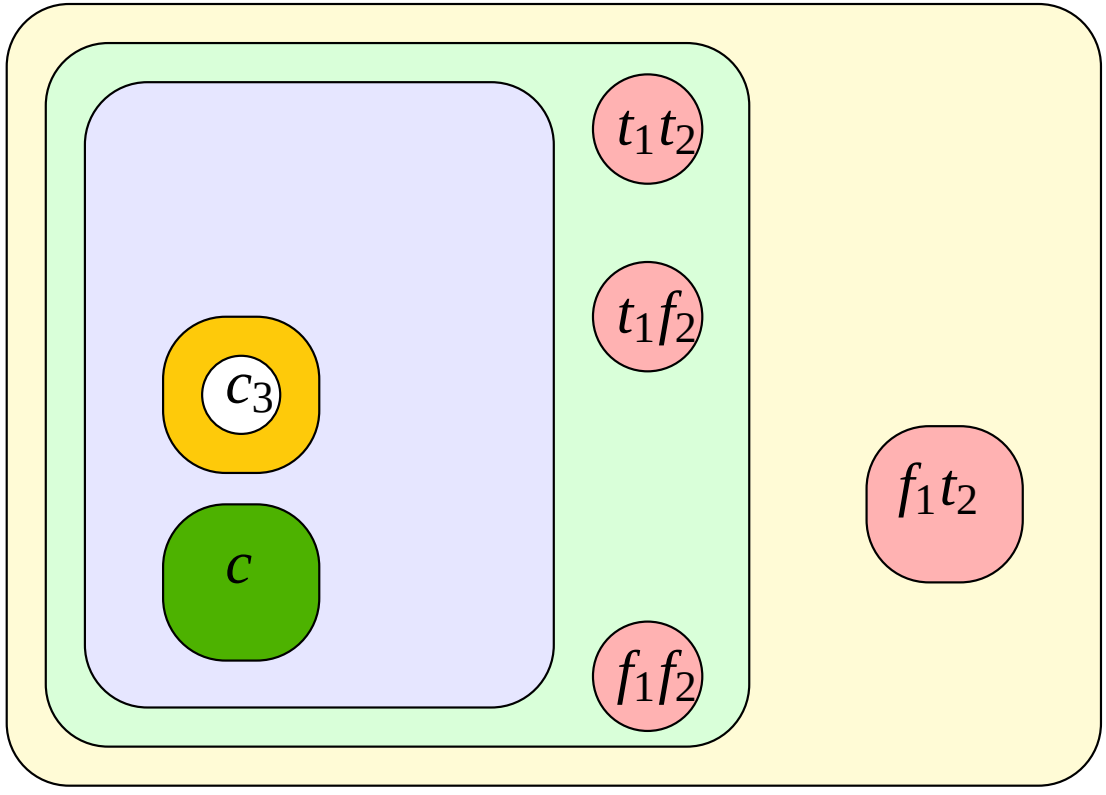

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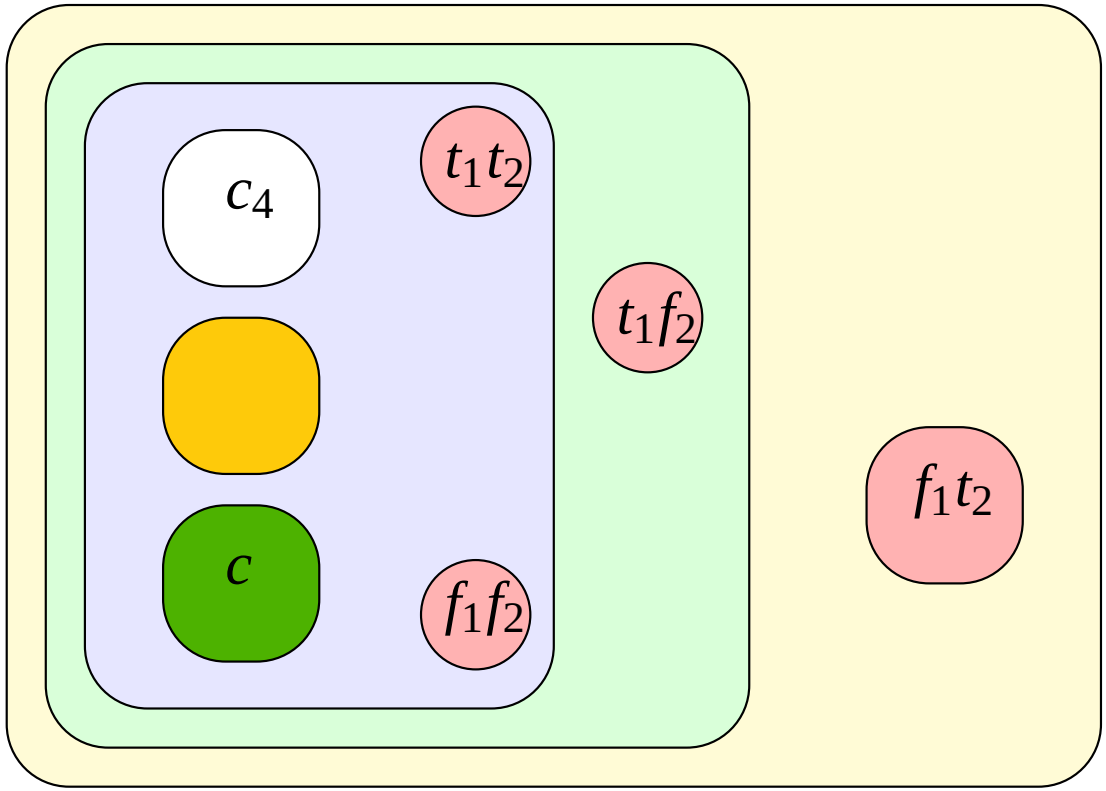

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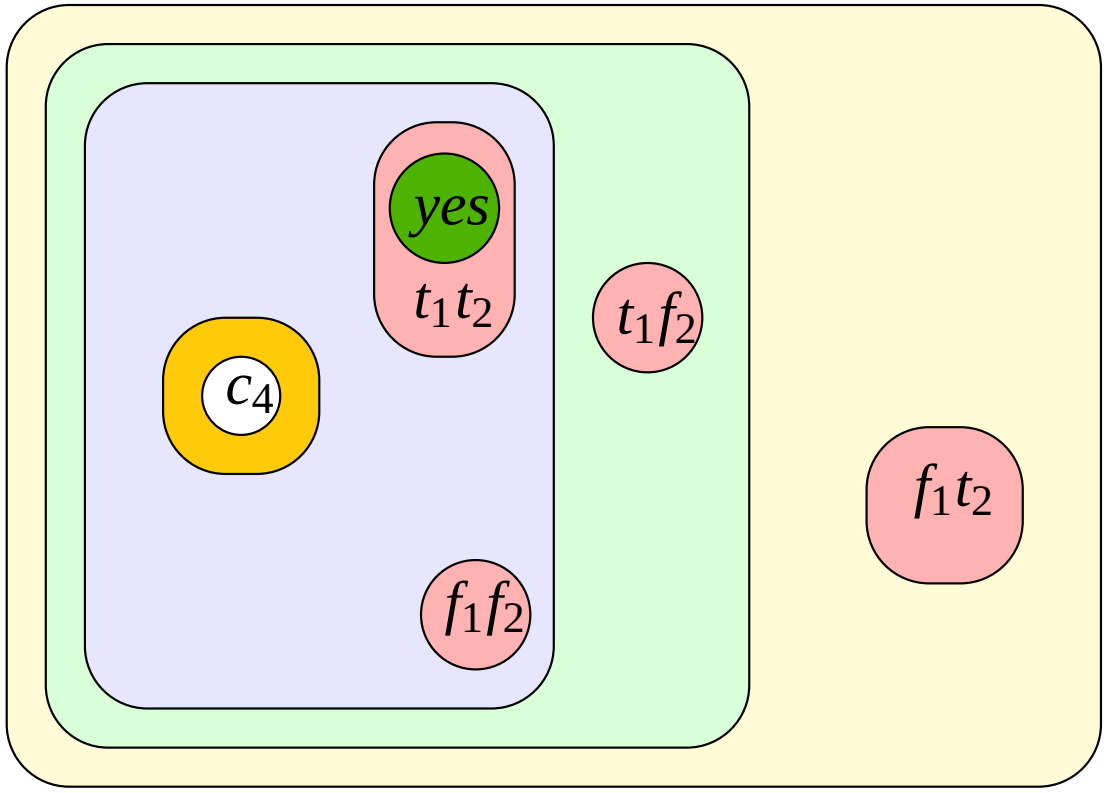

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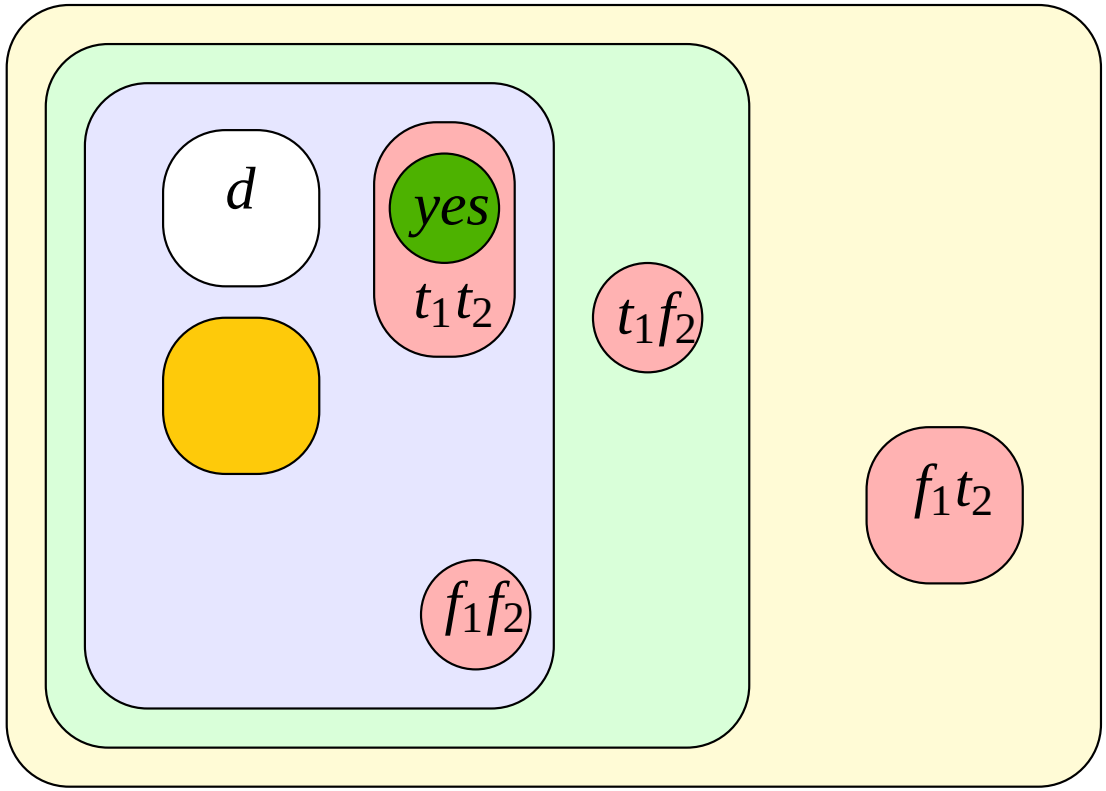

Applications

- biology, medicine, ecosystems
- computer science
- linguistics (modeling framework/parsing)
- optimization (membrane algorithms)
- economics

A typical application in biology/medicine

M. J. Pérez-Jiménez, F. J. Romero-Campero. A Study of the Robustness of the EGFR Signalling Cascade Using Continuous Membrane Systems. In *Mechanisms, Symbols, and Models Underlying Cognition. First International Work-Conference on the Interplay between Natural and Artificial Computation*, IWINAC 2005.

- 60 proteins, 160 reactions/rules
- reaction rates from literature
- results as in experiments

Implementation Efforts

Currently underway at *E. Keinan lab at Technion, Haifa*. Success would mean a huge boost to the area, and the development of *wet computers*

Open problems, research directions

- borderlines: universality/non-universality, efficiency/non-efficiency
  - local problems : the power of 1 catalyst, the role of polarizations, dissolution, etc
  - general problems : uniform versus semi-uniform, deterministic versus non-deterministic, complexity classes etc
- semantics (operational, behavioural)
- user friendly, flexible, efficient software for bio-applications
- implementations (electronic, bio-lab)

State of the art

- `http://ppage.psystems.eu`
- *Handbook of Membrane Computing*, Oxford University Press, 2010.

Across the world

Originated in Turku, Finland in Nov 1998. Major active groups

- Bucharest and Iaşi, Romania
- Budapest, Hungary
- Caltech and Santa Barbara, USA
- Edinburgh and Sheffield, UK
- Leiden, The Netherlands
- Madrid and Seville, Spain
- Milan and Verona, Italy
- Chennai and Mumbai, India
- Opava, Czech Republic
- Toyama and Waseda, Japan
- Vienna, Austria

THANKYOU